Soluții – clasa a VII-a

1. a)Evident, $n\neq 0$ si $n\neq 3$

   Pentru orice $n\in\mathbf{Z}$, numitorul raportului este număr par, iar numărătorul este impar, deci $\frac{2n^2-4n+3}{n^2-3n}\notin \mathbf{Z}$.

   b)Din $\frac{2n^2-4n+4}{n^2-3n}\in \mathbf{Z}$ => $\frac{2n^2-6n+2n+4}{n^2-3n}\in \mathbf{Z}$ =>$\frac{2(n^2-3n)}{n^2-3n}+\frac{2n+4}{n^2-3n}\in \mathbf{Z}$ => $\frac{2n+4}{n^2-3}\in \mathbf{Z}$ (1)

   =>$\frac{n(2n+4)}{n^2-3}\in.\mathbf{Z}$ =>$\frac{2n^2-6n+10n}{n^2-3}\in \mathbf{Z}$=>2+$\frac{10n}{n^2-3n}\in \mathbf{Z}$ =>$\frac{10n}{n^2-3n}\in \mathbf{Z}$ (2)

   Din (1) => $\frac{5(2n+4)}{n^2-3}\in \mathbf{Z}$ ⇔ $\frac{10n+20}{n^2-3n}\in \mathbf{Z}$ (3)

   şi , ţinând seama de (2) => $\frac{20}{n^2-3n}\in \mathbf{Z}$ => $n^2-3n\in D_{20}$

   Obținem $n\in\{1,2,4\}$

2. a) E(1)+E(2)+…+ $\frac{E(2006)}{2006}$ =(-1+2)+…+ $\frac{(-2005)+2006)}{2006}=\frac{1003}{2006}=\frac{1}{2}$

   b) n=2t, E(1)+…+E(2t)=t => 2007a, $a\in N$

   ⇨ N=4014a, $a\in N$, $a\in N^*$

   n=2t+1, E(1)+…+E(2t+1)=-t-1=-(t+1).

   t+1=2007a, $a\in N$

   ⇨ n=4014a-1, $a\in N^*$

   $n\leq 10000$ => $n\in\{4013,4014,8027,8028\}$

   c)k^2+E(n)=1 => E(n)=1-k^2<0 => n m impar

   n=2t+1 => -t-1=1-k^2 => t=k^2-2

Pentru k impar => $n\in\emptyset$

Pentru k par => n = 2 k^2 – 3

3. a) Din ΔCEB~ΔCMN => $\frac{EB}{MN}=\frac{CB}{CN}$ şi din ΔDAF~ΔDMN, => $\frac{AF}{MN}=\frac{DA}{DM}$. Dar $\frac{CB}{CN}=\frac{DA}{DM}$, => $\frac{EB}{MN}=\frac{AF}{MN}$ => EB=AF => EA=BF

b) Din ΔEBC~ΔMNC, => $\frac{EB}{MN}=\frac{CB}{CN}=\frac{CN+NB}{CN}=1+\frac{NB}{CN}=1+\frac{MN}{b}$ => $\frac{MN+a}{MN}=1+\frac{MN}{b}$.

$\frac{MN+a}{MN}=\frac{b+MN}{b}$ => $\frac{MN+a-MN}{MN}=\frac{b+MN-b}{b}$ => $\frac{a}{MN}=\frac{MN}{b}$ => $MN^2=ab$

4. a) ΔARC~ΔADB => $\frac{RC}{DB}=\frac{AC}{AB}$ => RC*AB=DB*AC.

   b) $DD_1\perp AC$ şi $DD_2\perp AB$ => $DD_1=DD_2$

$\left.\begin{array}{l} DD_2 \perp AB \\ ADB=90^0;\ AD=DD \end{array}\right\} \Rightarrow DD_2=\frac{1}{2}AB.$

$S_{ADC}=\frac{1}{2}\ AC*DD_2=\frac{1}{2}*\frac{1}{2}*AC*AB=\frac{1}{2}S_{ABC}=S_{ABR}$

c) (AB bisectoare => $\frac{MB}{MC}=\frac{AB}{AC}$

$\frac{EA}{EB}*\frac{MB}{MC}*\frac{FC}{FA}=S_{ADC}/S_{BDC}*AB/AC*S_{RCB}/S_{RAB}=S_{RCB}/S_{BDC}*AB/AC$

$=\frac{1}{2}RC*DR/4*DB*DR*AB/AC=RC*AB/DB*AC=1$ => AM,CD,BR coliniare